Generalized Penalized Weighted Least-Squares Reconstruction for Deblurred Flat-Panel CBCT

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Abstract—An increase in achievable spatial resolution would enable application of flat-panel detector cone-beam CT (FP-CBCT) in high-resolution applications, such as 3D breast and bone imaging. Improved reconstruction techniques exploiting more sophisticated models of noise and spatial resolution characteristics may provide such improvement. In this work, we propose a statistical model of measurement mean and covariance, with particular attention given to correlations in measurement data. An iterative reconstruction algorithm based on this model was applied to simulated and real experimental data to evaluate its performance compared to an uncorrelated model under various imaging conditions. Focal spot and detector blur of a CBCT testbench were measured and used to inform the model-based image reconstruction. Simulation studies demonstrate that incorporation of the correlated noise model yields improved reconstructions, especially in the presence of focal spot blur. Improved noise-resolution tradeoffs were quantified, and application to high-resolution imaging of trabecular bone was demonstrated.

Keywords—Spatial resolution, Cone-beam CT, Correlated noise, Model-based Reconstruction, Generalized Least-Squares Estimation

I. INTRODUCTION

Flat-panel cone-beam computed tomography (FP-CBCT) has found many applications in high spatial resolution imaging due to the large volumetric coverage and small detector pixels. Despite this advantage, the spatial resolution required for some applications remains just out of reach. For example, visualization of microcalcifications in CBCT mammography or highly detailed trabecular structure in quantitative CT of musculoskeletal extremities both stand to benefit from improvements beyond conventional spatial resolution limits. Current breast imaging solutions include projection mammography, tomosynthesis, and CBCT, with the requirement to detect microcalcifications at a scale of ~0.1 mm. Similarly in trabecular bone imaging, the gold standard is microCT which tends to have a very small field of view, making it difficult to apply to in vivo applications. In this paper, we seek to extend the high spatial resolution capabilities of FP-CBCT through careful modeling of system blur and correlated noise within a statistical reconstruction framework.

Typical model-based iterative reconstruction algorithms balance accuracy and complexity of the imaging system model based on desired resolution, noise properties, algorithm complexity, and execution time. The standard approach is to adopt a projector model with little or no blur and independent measurement noise. Such assumptions have been supported in part by work done by Hofmann et al. [1], which shows that modeling source blur in clinical CT systems does not significantly improve image quality for typical focal spot sizes (~0.5 mm) and detector pixels sizes (~0.6 mm). While this may be true for current diagnostic CT systems and current spatial resolution goals, there are a number of factors that suggest an advantage by way of improved system models for FP-CBCT. Such factors include 1) finer detector pixels (70-200 μm) in flat panels; 2) larger measurement blur due to light spread in the scintillator; 3) increased focal spot blur due to widespread use of inexpensive fixed anode x-ray tubes and compact geometries in dedicated FP-CBCT systems; and 4) the presence of significant noise correlation in FP-CBCT data due to light spread after the conversion of primary x-ray quanta to secondary light photons.

Model-based reconstruction approaches that account for system blur but without a correlated noise model have been previously developed. [2], [3] Similarly, sinogram restoration methods have also been proposed to correct for system blur [4] and noise correlation [5]. We propose a combination of sinogram restoration (deblurring) followed by iterative reconstruction, tracking the correlations through the deblur processing and accounting for them in the reconstruction step.

Flat-panel CBCT systems can be characterized by multiple sources of blur and noise. We consider blur due to an extended x-ray source and due to the detector scintillator, the latter of which adds correlations to the noise. Because noise correlation is a result of only one of these two sources of modeled blur, we explore the effects of varying the magnitude of these blurs in simulation. Understanding the scenarios that benefit from blur and correlation models will not only improve reconstructions on current FP-CBCT systems, but will help to guide new system design (e.g., geometry, focal spot size, and scintillator thickness).

In the following sections we present an overview of the reconstruction methodology. This approach is a refined version of the methods reported in [6] with a modified strategy for deblurring. A simulation investigation for various system designs with different magnitudes of source and detector blur follows. Subsequently, a brief description of the physical measurement of source and detector blur for a FP-CBCT testbench is presented. Finally, the measured blur models are incorporated within the proposed methodology and applied to FP-CBCT extremity data for trabecular bone imaging.

II. METHODS

A. Imaging System Model and Reconstruction

We consider a system model based on the first and second order statistics. Thus, the measurements are specified by the mean vector (\( \bar{y} \)) and the covariance matrix (\( K_p \)) which are functions of the attenuation (\( \mu \)), related by a forward projection operator (\( A \)), a gain operator (\( G \)), and the source and detector
scintillator blurs ($B_3$ and $B_1$, respectively). These models can be represented compactly in vector form as

$$\hat{y} = B_3 B_1 G \exp(-A\hat{\mu})$$  

$$K_y = B_3 D(B_1 G \exp(-A\hat{\mu}))(B_3^T + K_{ro})$$

where $K_{ro}$ denotes additive readout noise in the detector. (Note that source blur is approximated as a projection domain effect – implying that depth dependent magnification effects are small.)

Transforming measurements into line integral estimates and assuming Gaussian noise permits accommodation of a non-diagonal covariance matrix within a linear least-squares framework. However, such linearization requires an initial deblur operation. We introduce a “modified” deblur operator

$$C^{-1} \approx B_d^{-1} B_s^{-1}$$

which is approximately equal to the inverse of the source and detector blurs. Exact inversion may not be possible or desirable due to null spaces and noise magnification.

The line integral estimate and its covariance then become

$$l = -\log(G^{-1}C^{-1}y)$$

$$K_l \approx D \left\{ \frac{1}{(C^{-1}y)^2} \right\} C^{-1} K_y [C^{-1}]^T D \left\{ \frac{1}{(C^{-1}y)} \right\}$$

yielding the penalized weighted least-squares equation:

$$\hat{\mu} = \arg \min_{\mu} \| l - A\mu \|_{K_l^{-1}} + \beta R(\mu)$$

B. Practical Implementation

In this work, we use a quadratic penalty function, allowing equation (6) to be written in a closed form.

$$\hat{\mu} = [A^T K_l^{-1} A + \beta R^{-1}]^{-1} A^T K_l^{-1} l$$

We assume all blurs are shift invariant, allowing blur operations to be done in the Fourier domain. We choose the deblur operation in (3) so that it applies a binary mask ($M$) zeroing any nullled frequencies, approximated as the frequencies where the blur MTF is less than $10^{-2}$ (simulation) or $10^{-4}$ (testbench data).

$$C^{-1}x = F^{-1} \left( M \frac{F(x)}{F(B_s B_d)} \right)$$

where $F$ denotes the Fourier transform. Consequently:

$$C^{-1} \approx C^* x = F^{-1}(F(B_s B_d) M F(x))$$

which allows the inverse covariance matrix to be written as

$$K_l^{-1} \approx D[C^{-1}y][C^*]^T K_y^{-1} [C^*] D[C^{-1}y]$$

where $K_y$ is approximated as

$$K_y \approx B_d D[C^{-1}y] B_d^T + K_{ro}$$

The null space in $C^*$ forbids an inversion of $K_l$, but using iterative methods we may compute a generalized inverse and solve the likelihood equation within the unmasked subspace.[7]

A similar approach was used for null spaces in $B_d$ when inverting $K_y$ in simulation studies.

Solving of (7) can be divided into two parts, preprocessing and iterative steps. In the preprocessing step line integral estimates are computed from the measurements using the modified deblur, the inverse covariance matrix is applied, and the result is backprojected. The inverse in (7) is solved iteratively using the conjugate gradient method (100 iterations for simulated data, 300 for real data). Each iteration requires a convolusion with the penalty kernel, a forward projection, an application of the inverse covariance, and a backprojection. The application of the covariance matrix requires an application of the inverse of $K_y$, also via the conjugate gradient method (1000 iterations in the preprocessing step and 100 in each iteration of the iterative step, or until the residual decreased to 0).

C. Simulation Studies

Figure 1 shows the simulation phantom. Projection data were generated using a high resolution version of the phantom (25 µm voxels) onto a high resolution detector (1x7000 line of 35 µm pixels at 720 angles), which was then downsampled to 1x1750 at 720 angles. The data were reconstructed into a 1000x1000 grid of 0.1 mm voxels. Gaussian noise was added to the data to simulate correlated quantum noise and white readout noise. Noiseless data generation from line integrals is shown in (12), and noise injection is show in (13).

$$\hat{y} = B_s B_3 G \exp(-l)$$

$$y = \hat{y} + B_d N \left( 0, \sqrt{B_s B_3 G \exp(-l)} \right) + N(0, \sigma_{ro})$$

The variance of the readout noise was 3.5 photons². Gain was a constant 10⁶ photons.

Two reconstruction methods were studied: 1) using the full covariance matrix in equation (10); and 2) an uncorrelated covariance matrix composed of only diagonal elements:

$$K_L^{-1} = D[C^{-1}y] + D[\sigma_{ro}^2]$$

which represents an independent noise assumption. When this noise model is used, (7) is a traditional penalized least-squares reconstruction with deblurred measurements. The noise models were compared by plotting two resolution-variance curves from reconstructions with varying penalty strength.

Resolution was measured as an edge response to the disc in the phantom, and variance was approximated as the spatial variance of the data inside the blue ring shown in Figure 1. Additional FDK reconstructions were conducted with the cutoff frequency at Nyquist and no apodization.

To evaluate the importance of the two types of blurs, various blur scenarios were tested. There were two subsets of experiments where: 1) the total blur was kept constant, and the distribution of blur was altered from source dominated to detector dominated; and 2) the ratio of the blurs was kept constant, and the value of the total blur was varied. All blurs in the simulation study were Gaussian.

D. Testbench Characterization

In order to apply this algorithm to physical FP-CBCT data, we estimated the source and detector blurs on our x-ray testbench. This was done by imaging a tungsten edge, calculating the edge spread function (ESF), and then converting this to a modulation transfer function (MTF).[8] The scintillator
blur was assumed to be radially symmetric. The source blur is known to not be radially symmetric, so MTFs were taken in the (approximate) vertical, horizontal, and 45 degree directions. These results were compared with MTFs taken from a pinhole image of the source, acquired by placing a lead plate with a pinhole on the filter and using a geometry with a high magnification.

E. Physical FP-CBCT testbench data studies

Our testbench includes a Varian 4030CB detector and a Varian Rad-94 x-ray tube. A custom wrist phantom (The Phantom Laboratory, Greenwich, NY) was imaged with large (~1.1 mm) and small (~0.5 mm) focal spots. We used the large focal spot for testing our method, and the small focal spot to create a high resolution reference image using FDK. The data were reconstructed into a 210x600x600 volume of .15 mm voxels using the correlated and uncorrelated noise models with readout noise modeled as a constant 3.5 photons². These two iterative reconstruction methods were noise matched. Deblurred and non-deblurred data were also reconstructed using FDK. FDK reconstructions were done using a cutoff at Nyquist and no apodization.

III. RESULTS

Figure 2 shows resolution variance curves for simulated systems with different combinations of source and detector blur. Points positioned on the curve in the diagram all have the same total blur, while points on the straight line have the same ratio of source to detector blur. The correlated noise model equals or outperforms the uncorrelated noise model in all cases. When source blur is large there is a significant advantage to the correlated noise model whereas when source blur is minimal, the two methods are nearly equivalent. This is consistent with the fact that only the detector blur adds correlation and when source blur is absent, the deblurring operation whitens the noise, making the uncorrelated noise assumption accurate. When detector blur is absent, the data is uncorrelated and deblurring source blur adds off-diagonal values to the covariance matrix making the correlated noise model important. When the ratio of the blurs is kept constant and total blur...
Figure 5 shows real data reconstructions. Figure 5A is a high resolution reference image, with a zoom region (Figure 5B) and patch for variance estimation identified. A coarser resolution FDK reconstruction where most of the high resolution trabecular structure has been lost is shown in Figure 5C. Attempting to deblur (using the Gaussian approximations discussed previously) prior to applying FDK results in significant noise amplification (Figure 5D). In comparison, noise matched uncorrelated (Figure 5E) and correlated (Figure 5F) noise model reconstructions are also shown. Again, the correlated noise model achieves higher resolution - showing finer structures than the uncorrelated case at matched noise. Comparisons with the high resolution reference indicate that these fine structures are representative of true trabecular details.

IV. DISCUSSION

In this work we have presented a general framework for modeling both source and detector blur including correlated noise effects. Investigations in both simulations and in physical FP-CBCT data show that proper noise modeling can improve the noise-resolution trade-off. This advantage is most dramatic for systems with significant source blur, though deblurring and an accurate noise model outperforms conventional FBP in all cases considered.

This methodology has potential application in a variety of applications requiring high spatial resolution, including trabecular bone imaging and breast CBCT. Moreover, the studies presented here begin to characterize the improvements possible for specific system designs (e.g., varying amounts of source and detector blur). This may have potential impact in system design – for example, permitting use of larger focal spot x-ray sources (improved power requirements) and thicker scintillators (improved detection efficiency) – if the proposed reconstruction methodology is integrated into the system.

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